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## NUMBER THEORY WORKSHOP BY GANDA

University of Aix-Marseille,  
September 26 - September 29, 2023  
organised by Sary Drappeau ([sary-aurelien.drappeau@univ-amu.fr](mailto:sary-aurelien.drappeau@univ-amu.fr)) and Fabien Pazuki  
([fpazuki@math.ku.dk](mailto:fpazuki@math.ku.dk)),  
with the support of the CNRS (France),  
of the Department of Mathematical Sciences of Copenhagen University (Denmark),  
and of the Department of Mathematical Sciences of Aix-Marseille University (France).

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### Program

	Tuesday 26.09	Wednesday 27.09	Thursday 28.09	Friday 29.09
09:00-10:00		<b>Risager</b>		<b>Luca</b>
10:00-10:30		<i>Coffee break</i>		<i>Coffee break</i>
10:30-11:30		<b>Razakarino</b>		<b>Anni</b>
11:30-14:00	<i>Lunch</i>	<i>Lunch</i>	<i>Lunch</i>	<i>Lunch</i>
14:00-15:00	<b>Destagnol</b>		<b>Habilitation</b>	
15:00-15:30	<i>Coffee break</i>		<b>Sary</b>	
15:30-16:30	<b>Nordentoft</b>		<b>Drappeau</b>	

### RESEARCH TALKS

TIME: Tuesday 26.09, 14:00-15:00.

ROOM: Salle de séminaire 2, bâtiment Ancienne BU, Luminy.

SPEAKER: **Kevin Destagnol** (Univ. of Paris-Sud, [kevin.destagnol@math.u-psud.fr](mailto:kevin.destagnol@math.u-psud.fr)).

TITLE: *Sums of arithmetic functions over values of polynomials and applications to the Loughran-Smeets conjecture.*

ABSTRACT: I will explain how one can estimate sums of arithmetic functions over values of polynomials provided that the arithmetic functions is well-behaved in arithmetic progressions and that the number of variables of the polynomials is big enough. I will then give a few applications of this result to the Loughran-Smeets problem regarding the probability for a random algebraic variety among a family to admit a rational point. This is joint work with Efthymios Sofos and Leonard Hochfilzer.

TIME: Tuesday 26.09, 15:30-16:30.

ROOM: Salle de séminaire 2, bâtiment Ancienne BU, Luminy.

SPEAKER: **Asbjørn Nordentoft** (Univ. of Paris-Sud, [acnordentoft@outlook.com](mailto:acnordentoft@outlook.com)).

TITLE: *Equidistribution of  $q$ -orbits of closed geodesics*

ABSTRACT: A celebrated theorem of Duke states that the closed geodesics equidistribute on the modular curve when ordered by discriminant. In this talk I will explain a  $q$ -orbit analogue where the geodesics are ordered by “the left-lower entry”.

TIME: Wednesday 27.09, 9:00-10:00.

ROOM: Salle de séminaire 2, bâtiment Ancienne BU, Luminy.

SPEAKER: **Morten Risager** (Univ. of Copenhagen, [risager@math.ku.dk](mailto:risager@math.ku.dk)).

TITLE: *Counting and equidistribution over primes in hyperbolic groups.*

ABSTRACT: In the 1980ies Good proved various counting and equidistribution results in hyperbolic groups. We revisit some of these results for specific arithmetic groups and investigates what happens when we restrict to counting according to prime lengths. This is work in progress joint with Yiannis Petridis.

TIME: Wednesday 27.09, 10:30-11:30.

ROOM: Salle de séminaire 2, bâtiment Ancienne BU, Luminy.

SPEAKER: **Brice Razakinoro** (Univ. of Stellenbosch, [brice@aims.ac.za](mailto:brice@aims.ac.za)).

TITLE: *Some results on the bound of Siegel zeros for quadratic fields.*

ABSTRACT: We are interested in the explicit version of the paper of Goldfeld and Schinzel entitled *On Siegel's zero*. In this talk, we begin by reviewing some elementary definitions and background results that we need to examine this paper. We also give a survey on what we know explicitly about this type of zero, if it exists. The last part of the talk is a presentation of the key steps used in the article which might include some interaction with the audience.

TIME: Friday 29.09, 9:00-10:00.

ROOM: Salle de séminaire 2, bâtiment Ancienne BU, Luminy.

SPEAKER: **Florian Luca** (Univ. of Witwatersrand and MPI-SWS, [florian.luca@wits.ac.za](mailto:florian.luca@wits.ac.za)).

TITLE: *Twisted rational zeros of linearly recurrent sequences.*

ABSTRACT: Let  $(T_n)_{n \in \mathbb{Z}}$  be the Tribonacci numbers given by  $T_{n+3} = T_{n+2} + T_{n+1} + T_n$  with  $T_0 = 0, T_1 = T_2 = 1$ . Marques and Lengyel (2014) obtained a formula relating the 2-adic valuation of  $T_n$  with the 2-adic valuation of a linear function of  $n$  (which might be constant) according to the residue class of  $n$  modulo 32 and optimistically conjectured that a similar formula holds true for every prime  $p$ . In the first part of the talk, we will see that their conjecture was indeed far too optimistic; in particular, it fails for all but seven primes below 600. Along the way, we are led to consider a certain twisted rational zero of a linearly recurrent sequence. We prove that nondegenerate linearly recurrent sequences have only finitely many such twisted rational zeros, which may be seen as an extension of the Skolem-Mahler-Lech theorem. While our method is, in general, not effective, in some cases we can compute all such twisted rational zeros. In particular, returning to the Tribonacci sequence we show that its only twisted rational zeros are the integral ones  $-17, -4, -1, 0$  together with the rational non-integral ones  $1/3$  and  $-5/3$ .

This is joint work with Y. Bilu, J. Nieuwveld, J. Ouaknine and J. Worrell.

TIME: Friday 29.09, 10:30-11:30.

ROOM: Salle de séminaire 2, bâtiment Ancienne BU, Luminy.

SPEAKER: **Samuele Anni** (Univ. of Aix-Marseille, [samuele.anni@gmail.com](mailto:samuele.anni@gmail.com)).

TITLE: *Semistable elliptic curves over totally real fields*

ABSTRACT: In this talk, I will present a bound for the degree of isogenies between semistable elliptic curves over totally real fields. This bound is particularly helpful for studying Diophantine equations and other problems. The proof of the bound is based on previous work, joint with Samir Siksek. Moreover, I will show some results about elliptic curves with prime conductor over totally real fields and their isogeny classes (work in progress with Alyson Deines).

## HABILITATION SARY DRAPPEAU

TIME: Thursday 28.09, 14:00-16:00.

ROOM: Amphi 12, bâtiment B, Luminy.

TITLE: *Indépendance statistique et lois limites pour quelques objets arithmétiques.*

RÉSUMÉ DU MÉMOIRE: Ce mémoire présente les thèmes sur lesquels ont porté mes travaux de recherche depuis mon arrivée à l'université d'Aix-Marseille en 2015. Leur problématique commune est de mettre en évidence des comportements statistiques réguliers dans des familles d'objets arithmétiques naturels : les fonctions multiplicatives ou additives, et les valeurs centrales de certaines familles de fonctions  $L$ .

La première partie concerne une question centrale en théorie multiplicative des nombres: celle d'estimer la corrélation des valeurs  $f(n)$  et  $g(n+1)$ , où  $f$  et  $g$  sont deux fonctions multiplicatives, notamment lorsque l'une des

deux fonctions est la fonction nombre de diviseurs. Ce problème est naturellement lié à la répartition de certaines suites dans les progressions arithmétiques, et trouvent des applications à d'autres questions arithmétiques, par exemple les zéros de petite hauteur des fonctions  $L$  de Dirichlet. Les majorations de sommes d'exponentielles algébriques sont un outil crucial dans cette partie du mémoire.

La seconde partie concerne certaines fonctions  $f : \mathbb{Q} \rightarrow \mathbb{C}$  nommées par Zagier formes modulaires quantiques, caractérisées par certaines symétries analogues à celles des formes modulaires. Mes collaborations sur ce sujet ont consisté d'une part à établir ces relations de modularité quantiques dans certains cas : celui de tordues additives de fonctions  $L$  de Dirichlet, et celui de sommes de symboles de Pochhammer; et d'autre part à les utiliser pour en déduire, par des méthodes de systèmes dynamiques, l'existence de lois limites pour les valeurs de  $f$  aux nombres rationnels ordonnés par dénominateurs croissants.