Analytic number theory week by GANDA

University of Witswatersrand,
September 30 - October 4, 2019

organised by Florian Luca (florian.luca@wits.ac.za) and Fabien Pazuki (fpazuki@math.ku.dk),
with the support of the CNRS (France), of the Department of
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Program

COURSES	Monday 30.09	Tuesday 01.10
08:30-10:15	**Morten Risager**	**Kevin Destagnol**
10:15-10:30	*Coffee break*	*Coffee break*
10:30-12:15	**Asbjørn Nordentoft**	**Florian Luca**
12:15-13:45	*Lunch break*	*Lunch break*
13:45-15:30	**Augustine Munagi**	**Darlison Nyirenda**
15:30-15:50	*Coffee break*	*Coffee break*
15:50-17:35	**Sary Drappeau**	*+Student talks+*
18:00	*Social event*	
RESEARCH	Wednesday 02.10	Thursday 03.10
09:00-09:50	**Kevin Destagnol**	**Fabien Pazuki**
09:50-10:30	*Coffee break*	*Coffee break*
10:30-11:20	**Florian Luca**	**Morten Risager**
11:30-12:20	**Asbjørn Nordentoft**	
12:20-13:45	*Lunch break*	
13:45-14:35	**Sary Drappeau**	
14:45-18:05	*+Student talks+*	
18:30	*Conference dinner*	
STUDENT TALKS	Tuesday 01.10	Wednesday 02.10
14:45-15:15		**Linda Goba**
15:20-15:50		**Molatelo Olgar Rapudi**
15:50-16:20	**Mark Sias**	*Coffee break*
16:25-16:55	**Beaullah Mugwangwavari**	**Ifeanyi Patrick Iroanya**
17:00-17:30	**Siphephelo Ngubane**	**Damon Moodley**
17:35-18:05		**Faratiana Brice Razakarinoro**

Courses

Time: Monday 30.09, 08:30-10:15.
Room: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
Speaker: **Morten Risager** (Univ. of Copenhagen, risager@math.ku.dk).
Title: *The Gauss circle problem.*
Abstract: We will introduce the Gauss circle problem and explain some of the most basic techniques that goes into address it.

Time: Monday 30.09, 10:30-12:15.
Room: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
Speaker: **Asbjørn Nordentoft** (Univ. of Copenhagen, nordentoft@math.ku.dk).
Title: *A gentle introduction to modular forms.*
Abstract: I will give an introduction to modular forms starting from the very basics. In particular I will present Ramanujan's tau-function and Eisenstein series and present some applications to elementary number theory. If time permits I will give a short introduction to Hecke operators and L-functions of modular forms.

Time: Monday 30.09, 13:45-15:30.
Room: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
Speaker: **Augustine Munagi** (Univ. of Witswatersrand, augustine.munagi@wits.ac.za).
Title: *Algebraic structures in integer partitions.*
Abstract: A.O.L Atkin seems to be the first to describe an algebraic structure on the set of partitions. In a seminal paper (Quart. J. Math 17 (1966)) defines an infinite abelian group on the set of all partitions based on successive subpartitions obtained by deleting perpendicular angles from partition Ferrers graphs. In this talk we use the multiplicities of summands, in natural order 1, 2, 3, ..., to define a group structure on the set of partitions and show that, unlike the Atkin model, a wide class of known sets of partitions form well-behaved groups of partitions. We also propose a corresponding ring structure.

Time: Monday 30.09, 15:45-17:30.
Room: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
Speaker: **Sary Drappeau** (Univ. of Aix-Marseille, sary-aurelien.drappeau@univ-amu.fr).
Title: *Probabilistic questions in number theory and dynamics.*
Abstract: One of the main motivating theme in probability theory is to find "universal laws" arising from chaotic or complicated phenomena. The law of large numbers and the central limit theorems are the two prototypes of this. We will talk about occurrences of these questions in number theory and dynamical systems, and some of the methods involved.

Time: Tuesday 01.10, 08:30-10:15.
Room: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
Speaker: **Kevin Destagnol** (Univ. de Paris-Sud, kevin.destagnol@math.u-psud.fr).
Title: *Perron formula, convexity bounds for L-functions and adelic Poisson summation formula.*
Abstract: We will first motivate this talk by explaining why studying averages of multiplicative functions is of crucial importance in analytic number theory. We will then develop two very important tools building on complex analysis, namely Perron formula and the convexity bounds for L-functions, allowing to get estimates for the averages of such multiplicative functions by relating them to the analytic properties of the associated Dirichlet series. If time permits, we will in the last part of the talk present another powerful tool allowing to get very precise estimate for sums of "nice" functions over some discrete groups, namely Poisson summation formula. We will start by recalling the Poisson summation formula in its classical setting before explaining how one can obtain an adelic version of this formula.

Time: Tuesday 01.10, 10:30-12:15.
Room: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
Speaker: **Florian Luca** (Univ. of Witswatersrand and MPIM Bonn, florian.luca@wits.ac.za).
Title: *Solving Diophantine equations with linear forms in logarithms.*
Abstract: This is a self contained 2 hour course in which I will present the technology of linear forms in logarithms (complex and p-adic) and show how to use such techniques together with reduction methods in

order to solve exponential Diophantine equations. A few concrete examples will be worked out.

TIME: Tuesday 01.10, 13:45-15:30.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Darlison Nyirenda** (Univ. of Witswatersrand, darlison.nyirenda@wits.ac.za).
TITLE: *Gaussian polynomials and the Rogers-Ramanujan identities.*
ABSTRACT: We cover a few basic concepts about Gaussian polynomials and their relationship with some restricted partition functions. We then give an elementary proof of the original Rogers-Ramanujan identities involving polynomials of David Bressoud.

RESEARCH TALKS

TIME: Wednesday 02.10, 09:00-09:50.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Kevin Destagnol** (Univ. of Paris-Sud, kevin.destagnol@math.u-psud.fr).
TITLE: *Counting rational points of bounded height via the height zeta function.*
ABSTRACT: Over the past decades, analytic number theory has proven tremendously efficient in tackling problems coming from algebraic or arithmetic geometry. One particularly emblematic instance of this efficiency is the problem of studying the distribution of rational points on some classes of algebraic varieties.

Let V be an algebraic variety with infinitely many rational points. Then a famous conjecture of Manin and Peyre predicts a very precise asymptotic formula for the number of rational points on V of height (or "size") less than some bound B as B goes to $+\infty$.

The aim of this talk will be to explain how to use three very classical tools in analytic number theory, namely Perron's formula, Poisson summation formula and convexity bounds for certain L-functions in the critical strip in order to tackle Manin and Peyre conjecture in the case of algebraic varieties with an underlying group structure.

TIME: Wednesday 02.10, 10:30-11:20.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Florian Luca** (Univ. of Witswatersrand and MPIM Bonn, florian.luca@wits.ac.za).
TITLE: *Ternary Egyptian fractions.*
ABSTRACT: An Egyptian fraction with k summands is a representation of the form

$$\frac{a}{n} = \frac{1}{m_1} + \cdots + \frac{1}{m_k}$$

with positive integers $a, n, m_1, \ldots, m_k$. What is of interest is, given n, to count $A_k^*(n) = \{a : (a, n) = 1, a/n = 1/m_1 + \cdots + 1/m_k$ for some $m_1, \ldots, m_k\}$ as well as $A_k(n)$ which is the same as $A_k^*(n)$ except that without the corporality condition on a and n. In my talk, I will survey what is known about this problem for $k = 2$ and I will show that

$$x(\log x)^3 \ll \sum_{p \le x} A_3^*(p) \ll x(\log x)^5.$$

We believe the lower bound is closer to the truth. The proof uses sieve methods and some results of Elsholtz and Tao on average sum of divisors functions over arithmetic progressions. This is joint work with F. Pappalardi.

TIME: Wednesday 02.10, 11:30-12:20.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Asbjørn Nordentoft** (Univ. of Copenhagen, nordentoft@math.ku.dk).
TITLE: *On the distribution of central values of additive twists of modular L-functions and period polynomials.*
ABSTRACT: Mazur and Rubin set forth conjectures regarding the distribution of modular symbols, one of which predicted the asymptotic distribution to be normal. This conjecture was recently settled by Petridis and Risager using an approach initiated by Goldfeld in the 90's. In this talk we will discuss the higher weight analogues of their results (with modular symbols corresponding to weight 2 cusp forms). From a cohomological point of view the period polynomials seems like the natural generalization, but we will determine the asymptotic distribution

and show that it is not normal. Central values of higher weight cusps forms is another natural generalization and using an extension of the methods of Petridis and Risager we show that central values of additive twists indeed do follow a normal distribution (a different proof in the case of level 1 was given by Bettin and Drappeau using dynamical methods).

TIME: Wednesday 02.10, 13:45-14:35.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Sary Drappeau** (Univ. of Aix-Marseille, sary-aurelien.drappeau@univ-amu.fr).
TITLE: *Central values of additive twists of L functions via continued fractions.*
ABSTRACT: Central values of L functions are an important object of study in analytic number theory, since in many circumstances they carry information on natural arithmetic quantities associated to these L functions (primes, ranks of elliptic curves...). Additive twists of central values associated with modular forms have been shown by Petridis-Risager (2017) and Nordentoft (2018) to be distributed according to a Gaussian law. This talk will be about a recent work with Sandro Bettin (Genova), where we obtain another proof (for level 1 forms) using tools from dynamical systems and continued fractions. Time permitting, we will also discuss applications to other natural invariants (central values of the Estermann function, Dedekind sums, Kashaev invariants).

TIME: Thursday 03.10, 09:00-09:50.
ROOM: Wits Science Stadium Hall 3, West Campus.
SPEAKER: **Fabien Pazuki** (Univ. of Copenhagen, fpazuki@math.ku.dk).
TITLE: *Regulators of number fields and elliptic curves.*
ABSTRACT: We will start by discussing the relevance of regulators in the study of number fields and of elliptic curves defined over number fields. We will then describe non-trivial lower bounds on these regulators and some corollaries: in a recent collaboration with Pascal Autissier and Marc Hindry, we prove that up to isomorphisms, there are at most finitely many elliptic curves defined over a fixed number field, with Mordell-Weil rank and regulator bounded from above, and rank at least 4.

TIME: Thursday 03.10, 10:30-11:20.
ROOM: Law Building Room 144, West Campus.
SPEAKER: **Morten Risager** (Univ. of Copenhagen, risager@math.ku.dk).
TITLE: *The hyperbolic circle problem.*
ABSTRACT: The hyperbolic circle problem is harder than the Euclidian circle problem, and the best known error estimates are in a precise sense - which we will explain - much weaker than what has been proved towards conjectural bound in the Gauss circle problem. We will explore different averages – both of generic and arithmetic significance – which improves on the error terms in the hyperbolic problem.

STUDENT TALKS

TIME: Tuesday 01.10, 15:50-16:20.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Mark Sias** (Univ. of Witswatersrand, msias@uj.ac.za).
TITLE: *On members of Lucas sequences which are products of factorials.*
ABSTRACT: Here, we show that if $\{U_n\}_{n\geq 0}$ is a Lucas sequence, then the largest n such that $|U_n| = m_1!m_2!\cdots m_k!$ with $1 < m_1 \leq m_2 \leq \cdots \leq m_k$ satisfies $n < 2.1 \times 10^6$.It is expected that most, if not all, the elements of the presentation would be intelligible to an audience with even a smattering of Number Theory at undergraduate level.

TIME: Tuesday 01.10, 16:25-16:55.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Beaullah Mugwangwavari** (Univ. of Witswatersrand, 712040@students.wits.ac.za).
TITLE: *A review of Fu and Sellers map.*
ABSTRACT: H. Gupta, J. Sellers and M. R. Kanna bijectively proved that the number of partitions of n wherein even multiplicities of the parts are less than $2m$ and odd multiplicities are at least $2r+1$ and at most $2(m+r)-1$

is equal to the number of partitions of n in which parts are either odd and congruent to $2r+1 \pmod{4r+2}$ or even and not congruent to $0 \pmod{2m}$. In this talk, we shall show that applying Glaisher or Sylvester map to Sellers' map results in the bijective mappings of Kanna and Gupta, respectively.

TIME: Tuesday 01.10, 17:00-17:30.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Siphephelo Ngubane** (Univ. of Witswatersrand, 600507@students.wits.ac.za).
TITLE: *Conjugation of overpartitions and applications of over q-binomial coefficients.*
ABSTRACT: We discuss conjugation method for overpartitions and give the generating function for self-conjugate overpartitions. We then discuss an overpartition analogue of Gaussian polynomials (or q-binomial coefficients) which is defined as the generating function for overpartitions fitting inside an m x n rectangle. These are the over Gaussian polynomials (or over q-binomial coefficients). We derive the generating function for overpartitions fitting inside an n x n rectangle and deduce a new summative generating function for ordinary overpartitions. Lastly, we propose a solution to a Monotonicity conjecture of Dousse-Kim (J. Combin. Theory Ser. A, 2018)".

TIME: Wednesday 02.10, 14:45-15:15.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Linda Goba** (Univ. of Witswatersrand, 712639@students.wits.ac.za).
TITLE: *Compositions with relatively prime summands.*
ABSTRACT: Integer compositions have been studied for over 125 years, with the majority of new research dating back to the 1960's. Several authors have since studied compositions with restrictions on the sizes of the summands or on their order of arrangements. In this talk I will discuss compositions whose summands have been restricted to those that are relatively prime. We will also place a further restriction wherein the parts of the compositions are pairwise relatively prime. Moreover, I will consider special cases in which the compositions with relatively prime parts are represented as a unique product of primes, deriving much simple results. Then, if time permits I will discuss compositions wherein several copies of an integer part are allowed and how such compositions relate to our initial restriction.

TIME: Wednesday 02.10, 15:20-15:50.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Molatelo Olgar Rapudi** (Univ. of Witswatersrand, 942365@students.wits.ac.za).
TITLE: *On certain restricted integer partitions.*
ABSTRACT: In this talk we generalise a theorem of J.J Sylvester that relates self-conjugate partitions to partitions into distinct odd parts and give a geometric interpretation of the bijection for the theorem and/or give a new partition-theoretic interpretation of two Rogers-Ramanujan identities, once considered by A.K Agarwal (CRC press, 1st. ed., 2017).

TIME: Wednesday 02.10, 16:25-16:55.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Ifeanyi Patrick Iroanya** (Univ. of Witswatersrand, 1347143@students.wits.ac.za).
TITLE: *On parameters of binary cyclic codes.*
ABSTRACT: In this talk, we consider cyclic codes defined over finite fields of characteristic 2. Our goal is to determine parameters of certain cyclic codes constructed via the classical code method and also deduce the associated weight enumerator polynomials.

TIME: Wednesday 02.10, 17:00-17:30.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Damon Moodley** (Univ. of Witswatersrand, 747011@students.wits.ac.za).
TITLE: *Composite integers whose sum of prime factors is prime.*
ABSTRACT: In this talk we give upper an lower bounds on the counting function of the composite positive integers $n \leq x$ such that the sum of the prime factors of n is a prime. The proof uses sieve methods.

TIME: Wednesday 02.10, 17:35-18:05.
ROOM: CoE-MaSS Seminar Room, 1st floor, TW Kambule Mathematical Sciences Building, West Campus.
SPEAKER: **Faratiana Brice Razakarinoro** (Stellenbosch Univ., brice@aims.ac.za).
TITLE: *Explicit bound on Siegel zeros of imaginary quadratic fields.*
ABSTRACT: We consider the Dirichlet L-functions associated with characters defined by Kronecker's symbol.

It is known that these L-functions might have zeros very close to 1. These hypothetical zeros are called Siegel zeros. In 1975, Goldfeld and Schinzel proved an effective lower bound for the distance of the Siegel zeros from 1. In the thesis, we made Goldfeld-Schinzel's bound explicit for the case of primitive odd characters. In this presentation, I will first talk about imaginary quadratic fields, which are naturally related to primitive odd characters, then I will mention the steps in the proof of the result. I will also talk about a potential application of the explicit bound concerning the size of the torsion subgroup of a CM elliptic curve.