

NUMBER THEORY WEEK BY GANDA

University of Witwatersrand,
 October 28 - November 1, 2024
 organised by Augustine Munagi (Augustine.Munagi@wits.ac.za) and Fabien Pazuki (fpazuki@math.ku.dk),
 with the support of the CNRS (France),
 of the Department of Mathematical Sciences of Copenhagen University (Denmark),
 of the CoE-MaSS and of the University of Witwatersrand (South Africa).

We dedicate this conference to the 102th anniversary of the University of Witwatersrand and to the 152th anniversary of the Société Mathématique de France! Happy birthday!

PROGRAM

COURSES	Monday 28.10	Tuesday 29.10
09:00-10:30		Jiang Zeng
10:30-10:45		<i>Coffee break</i>
10:45-12:15		Richard Griffon
12:15-13:45	<i>Lunch</i>	<i>Lunch</i>
13:45-15:15	Richard Griffon	Marion Le Gonidec
15:15-15:30	<i>Coffee break</i>	<i>Coffee break</i>
15:30-17:00	Marion Le Gonidec	Jiang Zeng
17:30	<i>Social event</i>	
RESEARCH	Wednesday 30.10	
09:00-09:50	Florian Luca	
10:00-10:50	Augustine Munagi	
10:50-11:10	<i>Coffee break</i>	
11:10-12:00	Jiang Zeng	
12:00-13:30	<i>Lunch</i>	
13:30-14:20	Richard Griffon	
14:20-14:40	<i>Coffee break</i>	
14:40-17:20	+STUDENT TALKS+	
18:30	<i>Conference dinner</i>	
STUDENT TALKS		Wednesday 30.10
14:40-15:10		Andile Ngobeni
15:20-15:50		Faith Shadow Zottor
15:50-16:10		<i>Coffee break</i>
16:10-16:40		Manape Makoko Campbell
16:50-17:20		Beullah Mugwangwavari
18:30		<i>Conference dinner</i>

COURSES

TIME: Monday 28.10, 13:45-15:15.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Richard Griffon** (Univ. of Clermont Auvergne, richard.griffon@uca.fr).

TITLE: *The Brauer-Siegel Theorem: ninety years of developments.*

ABSTRACT: (Part 1) The Brauer-Siegel theorem stands as a cornerstone of twentieth-century number theory. It describes the asymptotic growth of the class number and the regulator of units of number fields in terms of their discriminants. This course is an invitation to explore the theorem's statement, its history, as well as some of the developments it has inspired in recent years. We will begin by reviewing the essential concepts from algebraic number theory and introducing key tools, including zeta functions of number fields and their basic properties. Building on this foundation, we will outline a modern proof of the Brauer-Siegel theorem in its classical formulation. I will highlight the upshots of this contemporary approach compared to the original arguments by Siegel (1935) and Brauer (1947). Next, we will introduce the Generalized Brauer-Siegel conjecture, as suggested in a foundational paper by Tsfasman and Vladut (2002), and discuss our current knowledge about this conjecture. This last section will provide insights into recent advances in this active area, and will serve as an extended introduction to my research talk.

TIME: Monday 17.10, 15:30-17:00.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Marion Le Gonidec** (Univ. of La Réunion, marion.le-gonidec@univ-reunion.fr).

TITLE: *Links between representations and properties of real numbers.*

ABSTRACT: (Part 1) The usual ways to represent a real number α of $[0, 1[$ is using its representation in a fixed integer base $b \geq 2$, i.e. by a sequence $(\alpha_n)_{n \geq 1}$ with $\alpha_n \in \{0, \dots, b-1\}$ or, maybe less used, using its continued fraction expansion, i.e. by a sequence $(a_n)_{n \geq 1}$ of natural integers, possibly unbounded. This course will try to investigate known and conjectured links between the properties (rationality/algebraicity/transcendence & diophantine approximation properties) of a real number and the "complexity" of its representations. At first sight, Part 1 should concern integer base representations and Part 2 should concern continued fraction expansions... but it is not so clear. We will have to navigate through both!

TIME: Tuesday 29.10, 09:00-10:30.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Jiang Zeng** (Univ. Claude Bernard Lyon 1, zeng@math.univ-lyon1.fr).

TITLE: *Some positivity problems in combinatorics and analysis.*

ABSTRACT: (Part 1) Moments of orthogonal polynomials: We first review the combinatorial theory of continued fractions and orthogonal polynomials. In a 1980 paper Flajolet demonstrated that the coefficients in the Taylor expansion of generic Stieltjes-type or Jacobi-type continued fractions can represent generating polynomials for Dyck or Motzkin paths, respectively, with height-dependent weights. The combinatorial interpretations of the moments and linearization coefficients of classical orthogonal polynomials will be given.

TIME: Tuesday 29.10, 10:45-12:15.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Richard Griffon** (Univ. of Clermont Auvergne, richard.griffon@uca.fr).

TITLE: *The Brauer-Siegel Theorem: ninety years of developments.*

ABSTRACT: (Part 2) The Brauer-Siegel theorem stands as a cornerstone of twentieth-century number theory. It describes the asymptotic growth of the class number and the regulator of units of number fields in terms of their discriminants. This course is an invitation to explore the theorem's statement, its history, as well as some of the developments it has inspired in recent years. We will begin by reviewing the essential concepts from algebraic number theory and introducing key tools, including zeta functions of number fields and their basic properties. Building on this foundation, we will outline a modern proof of the Brauer-Siegel theorem in its classical formulation. I will highlight the upshots of this contemporary approach compared to the original arguments by Siegel (1935) and Brauer (1947). Next, we will introduce the Generalized Brauer-Siegel conjecture.

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SPEAKER: **Marion Le Gonidec** (Univ. of La Réunion, marion.le-gonidec@univ-reunion.fr).

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SPEAKER: **Jiang Zeng** (Univ. Claude Bernard Lyon 1, zeng@math.univ-lyon1.fr).

TITLE: *Some positivity problems in combinatorics and analysis.*

ABSTRACT: (Part 2) Total Positivity of Matrices: We discuss conditions under which certain matrices, which are constructed from sequences of combinatorially significant polynomials, exhibit a property called total positivity. It highlights a connection between this subject and the combinatorial theory of continued fractions.

RESEARCH TALKS

TIME: Wednesday 30.10, 09:00-09:50.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Florian Luca** (Univ. of Stellenbosch, fluca@sun.ac.za).

TITLE: *On a question of Douglass and Ono.*

ABSTRACT: It is known that the partition function $p(n)$ obeys Benford's law in any integer base $b \geq 2$. A similar result was obtained by Douglass and Ono for the plane partition function $PL(n)$ in a recent paper. In their paper, Douglass and Ono asked for an explicit version of this result. In particular, given an integer base $b \geq 2$ and string f of digits in base b they asked for an explicit value $N(b, f)$ such that there exists $n \leq N(b, f)$ with the property that $PL(n)$ starts with the string f when written in base b . In my talk, I will present an explicit value for $N(b, f)$ both for the partition function $p(n)$ as well as for the plane partition function $PL(n)$.

TIME: Wednesday 30.10, 10:00-10:50.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Augustine Munagi** (Univ. of Witwatersrand, augustine.munagi@wits.ac.za).

TITLE: *Coming to terms with perfect partitions.*

ABSTRACT: A partition of a positive integer n is said to be perfect if the summands contain exactly one partition of every positive integer less than n . A partition is double-perfect if the summands contain two partitions of every integer between 2 and $n - 2$. Both perfect and double-perfect partitions are known to be enumerated by ordered factorization functions. The seed for this work was sowed during the 2022 GANDA at Wits. In the present talk we will report on progress towards an inclusive formula for the number of generalized double-perfect partitions as well as a conclusive enumeration result for Triple-perfect partitions. Then we suggest some directions for further research.

TIME: Wednesday 30.10, 11:10-12:00.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Jiang Zeng** (Univ. Claude Bernard Lyon 1, zeng@math.univ-lyon1.fr).

TITLE: *Some positivity problems in combinatorics and analysis.*

ABSTRACT: (Part 3, research) Gamma-Positivity of variations of Eulerian polynomials: Gamma-positivity is a property that polynomials with symmetric coefficients may have, which directly implies their unimodality. After a brief overview of the definitions and known facts, we will present some recent refinements of this property for variants of Eulerian polynomials.

TIME: Wednesday 30.10, 13:30-14:20.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Richard Griffon** (Univ. of Clermont Auvergne, richard.griffon@uca.fr).

TITLE: *New cases of the generalized Brauer-Siegel Theorem.*

ABSTRACT: Given an infinite sequence S of number fields one may wonder about the asymptotic behavior, as K runs through S , of the product of the class number of K by its regulator of units (in terms of the discriminant of K). The classical Brauer-Siegel theorem answers this question when the number fields in S have bounded degree. In the early 2000's, Tsfasman and Vladut suggested the Generalized Brauer-Siegel conjecture, GBS for short, which would answer this question for much more general sequences. Their conjecture, which would follow from GRH, is known to hold in a handful of situations: for instance, when the number fields in the sequence are almost normal over \mathbb{Q} (Tsfasman, Vladut, Zykin and Lebacque). In a very recent work with Philippe Lebacque and Gaël Rémond, we prove that GBS unconditionally holds in many more instances. As I will explain in the talk, the main ingredients in our proof are the introduction of the notion of Galois complexity of a number field, and a new "zero descent" principle. If time permits, I will exhibit a few concrete examples for which GBS is now proved.

STUDENT TALKS

TIME: Wednesday 30.10, 14:40-15:10.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Andile Ngobeni** (Univ. of Witwatersrand).

TITLE: *Generating functions and partitions identities.*

ABSTRACT: This presentation will discuss the application of generating functions to the study of partition identities, that is, statements that assert the equality of the cardinalities of two or more sets of integer partitions. Partition theory belongs to the intersection of number theory and combinatorics and relies on the extensive combinatorial and analytical tools offered by both areas of mathematics. We will focus on the use of generating functions to prove partition identities, and to explore the discovery of new partition identities. We seek to provide more elegant and more inclusive proofs of classical partition identities by means of generating functions.

TIME: Wednesday 30.10, 15:20-15:50.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Faith Shadow Zottor** (Univ. of Witwatersrand).

TITLE: *Common values of two k -generalized Pell sequences.*

ABSTRACT: Let $k \geq 2$ and let $(P_n^{(k)})_{n \geq 2-k}$ be the k -generalized Pell sequence defined by

$$P_n^{(k)} = 2P_{n-1}^{(k)} + P_{n-2}^{(k)} + \cdots + P_{n-k}^{(k)}$$

for $n \geq 2$ with initial conditions

$$P_{-(k-2)}^{(k)} = P_{-(k-3)}^{(k)} = \cdots = P_{-1}^{(k)} = P_0^{(k)} = 0, P_1^{(k)} = 1.$$

In this study, we look at the equation $P_n^{(k)} = P_m^{(l)}$ and show that it has no nontrivial solutions.

TIME: Wednesday 30.10, 16:10-16:40.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Manape Makoko Campbell** (Univ. of Witwatersrand).

TITLE: *On the Euler function of linearly recurrent sequences.*

ABSTRACT: Let $\phi(m)$ and $\sigma(m)$ be the Euler phi function at m and the sum of divisors function at m , respectively. Let $(U_n)_{n \geq 1}$ be any non-degenerate linearly recurrent sequence of integers whose general term is up to sign not a polynomial in n . In this talk, we show that the inequalities

$$\phi(|U_n|) \geq |U_{\phi(n)}| \quad \text{and} \quad \sigma(|U_n|) \leq |U_{\sigma(n)}|$$

hold for almost all natural numbers. In fact, we show that the set of $n \leq x$ for which the above inequalities fail is of cardinality $O(x/\log x)$. This is a recent result which was proved in collaboration with F. Luca.

TIME: Wednesday 30.10, 16:50-17:20.

ROOM: Fac-Science-MSB-UG-Seminar Room, UG floor, TW Kambule Mathematical Sciences Building, West Campus.

SPEAKER: **Beaullah Mugwangwavari** (Univ. of Witwatersrand).

TITLE: *Modulo 2 congruences for partitions with initial repetitions.*

ABSTRACT: George Andrews defined a partition with initial k -repetitions as one in which if j appears at least k times, all positive integers less than j appear at least k times. Motivated by these partitions, we derive parity formulas for several functions for this class of partitions. This is work in collaboration with Darlison Nyirenda.