ANR Flair

IRN MaDeF

Diophantine Geometry and L-functions: Hindry 65

Université de Bordeaux,
September 26 – 30, 2022
organised by Philippe Lebacque, Fabien Pazuki, Cecília Salgado.

Program

	Mon 26.09	Tue 27.09	Wed 28.09	Thu 29.09	Fri 30.09
09:00-09:30	*Registration*				
09:30-10:20	**Autissier**	**Ullmo**	**Bertrand**	**Desjardins**	**Gajda**
10:20-10:50	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*	*Coffee break*
10:50-11:40	**Brunault**	**Cadoret**	**Fischler**	**Pacheco**	**Breuer**
11:40-14:00	*Lunch break*	*Lunch break*	*Lunch break*	*Lunch break*	*Lunch break*
14:00-14:50	**Gasbarri**	**Cantoral**	Excursion:	**Maire**	**Griffon**
14:50-15:20	*Coffee break*	*Coffee break*	Bus	*Coffee break*	
15:20-16:10	**Silverman**	**Lombardo**	to	**Tsfasman**	
16:20-17:10		*Group picture*	Saint	**David**	
18:30	Cocktail		Emilion		
19:30				Dinner	

Abstracts

TIME: Monday 26, 9:30-10:20.
ROOM: Salle de conférence.
SPEAKER: **Pascal Autissier** (Bordeaux, France).
TITLE: *Distribution of the maximum of partial exponential sums.*
ABSTRACT: In analogy with character sums, I investigate in this talk the distribution of the maximum of partial exponential sums. More precisely, in a joint work with Bonolis and Lamzouri, we obtain precise uniform estimates for the distribution function of this maximum in a near optimal range, under a symmetry assumption. Proofs use analytic and probabilistic tools, as well as deep ingredients from algebraic geometry.

TIME: Monday 26, 10:50-11:40.
ROOM: Salle de conférence.
SPEAKER: **François Brunault** (Lyon, France).
TITLE: *K_4 of modular curves and L-functions of modular forms.*
ABSTRACT: I will explain a new construction of elements in the algebraic K_4 group of modular curves. The Beilinson conjectures predict that the regulators of these elements are related to L-values of modular forms of weight 2 at $s = 3$. Numerical experiments in PARI/GP lead us to conjecture that these elements are proportional to the elements defined by Beilinson using the Eisenstein symbol. An important ingredient in the construction is the S-unit equation for modular curves, where S is the set of cusps.

TIME: Monday 26, 14:00-14:50.
ROOM: Salle de conférence.
SPEAKER: **Carlo Gasbarri** (Strasbourg, France).
TITLE: *Deformation of two dimensional orbifolds and almost lifting of projective surfaces.*
ABSTRACT: In 1961, J. P. Serre gave the first example of smooth projective variety in positive characteristic which cannot be lifted in characteristic zero. After this many new examples have been proposed. In particular, we know many examples of smooth projective surfaces which are not liftable. On the other side, there are important classes of varieties which can be lifted: curves, abelian varieties, K3 surfaces... Since every variety is birational to an hypersuface of the projective space and this is always liftable, we see that "liftability" is not a birational propriety. Achinger and Zdanovicz even proposed an example of smooth, rational threefold which cannot be lifted. On the other side, Liedtke and Satriano proved that if a smooth projective surface is liftable, then every smooth surface birational to it is liftable too. After a partial overview of the topic, we will explain the main lines of the proof of the following

THEOREM: *Every projective surface in characteristic different from two and three is birational to a* normal *projective surface which can be lifted in characteristic zero.*

If time allows, we will present a list of (attackable) problems we can deduce from this circle of ideas.

TIME: Monday 26, 15:20-16:10.
ROOM: Salle de conférence.
SPEAKER: **Joseph Silverman** (Providence, USA).
TITLE: *Fourier Expansions and Non-negative Averages of Local Heights on Abelian Surfaces over Local Fields.*
ABSTRACT: Let K be a complete local field, and let E/K be an elliptic curve having split multiplicative reduction. The local canonical height on $E(K)$ is the sum of a non-negative intersection theory part and the value of the periodic Bernoulli polynomial $\mathbb{B}_2(T) = T^2 - T + \frac{1}{6}$. Hindry and I used the Fourier expansion of $\mathbb{B}_2(T)$ and its weighted averages to prove lower bounds for canonical heights in the setting of conjectures of Lang and Lehmer. In this talk I will present an analogous 2-variable periodic Bernoulli polynomial $\mathbb{B}_2(X, Y)$ associated to the local canonical height on an abelian surface A/K having completely split multiplicative reduction, including a formula for the Fourier expansion of $\mathbb{B}_2(X, Y)$ and some discussion on how to obtain non-negative weighted averages. This is a joint work with Nicole Looper.

TIME: Tuesday 27, 9:30-10:20.
ROOM: Salle de conférence.
SPEAKER: **Emmanuel Ullmo** (IHES, France).
TITLE: *Manin-Mumford in arithmetic pencils.*
ABSTRACT: We will describe a refinement of Manin–Mumford (Raynaud's Theorem) for abelian schemes over some ring of integers. Torsion points are replaced by special 0-cycles, that is reductions modulo some, possibly varying, prime of Galois orbits of torsion points. There is a flat/horizontal part and a vertical one. The irreducible components of the flat part are given by the Zariski closure, over the integers, of torsion cosets of the generic fibre of the abelian scheme. The vertical components are given by translates of abelian subvarieties, which "come from characteristic zero". The starting point of the proof is the "Lang–Hindry" method for Manin–Mumford. This is a joint work with Gregorio Baldi and Rodolphe Richard.

TIME: Tuesday 27, 10:50-11:40.
ROOM: Salle de conférence.
SPEAKER: **Anna Cadoret** (Paris, France).
TITLE: *Ghosts and family of abelian varieties with a common isogeny factor.*
ABSTRACT: Let S be a smooth, geometrically connected variety over a field k, with generic point η, $A \to S$ an abelian scheme over S and $\mathfrak{A}$ an abelian variety over k. We investigate the following local-global principle $\mathrm{P}(k)$: Assume that for every closed point s in S, $\mathfrak{A}$ is a geometric isogeny factor of A_s. Then $\mathfrak{A} \times_k \eta$ is a geometric isogeny factor of A_η. When k contains an infinite finitely generated field, (a stronger variant of) $\mathrm{P}(k)$ always holds. The interesting case in when k is finite. In that case, we construct a complete obstruction to $\mathrm{P}(k)$, which we call the ghost of $A \to S$ and which actually exists for an arbitrary motive. In particular, this enables us to exhibit abelian schemes (constructed by Bueltel using deformation methods or by Katz, using Naive Fourier transforms) for which $\mathrm{P}(k)$ fails.

This is a joint work with Akio Tamagawa.

TIME: Tuesday 27, 14:00-14:50.
ROOM: Salle de conférence.
SPEAKER: **Victoria Cantoral Farfan** (Göttingen, Germany).
TITLE: *Some remarks on the component group of the Sato–Tate group.*
ABSTRACT: The famous Sato–Tate conjecture for elliptic curves defined over a number field (without complex multiplication) predicts the equidistribution of Frobenius traces with respect to the Haar measure of the corresponding Sato–Tate group under the trace map. This conjecture has already been generalized for higher-dimensional abelian varieties, $K3$ surfaces, and pure motives of odd weight. It seems then natural to study in detail the Sato–Tate group to tackle the generalized Sato–Tate conjecture. During this talk, we are going to introduce the abovementioned conjecture, and subsequently, we will discuss the component group of the Sato–Tate group associated with abelian varieties defined over a number field of arbitrary dimension. This is joint work with Grzegorz Banaszak.

TIME: Tuesday 27, 15:20-16:10.
ROOM: Salle de conférence.
SPEAKER: **Davide Lombardo** (Pisa, Italy).
TITLE: *Torsion bounds for a fixed abelian variety and varying number field.*
ABSTRACT: Let A be an abelian variety over a number field K. The question I will discuss in this talk is that of bounding the size of the torsion subgroup of $A(L)$, as L ranges over the finite extensions of K. In particular, one can ask for the optimal exponent β_A such that for all $\varepsilon > 0$ there is a constant $C_{A,\varepsilon}$ such that

$$\#A(L)_{\mathrm{tors}} \leq C_{A,\varepsilon}[L : K]^{\beta_A+\varepsilon}$$

for all finite extensions L/K. Hindry and Ratazzi have formulated a precise conjecture in this direction: the value of β_A should be given by a simple formula involving the Mumford-Tate group of A (more precisely, of its geometrically simple isogeny factors). They also proved this conjecture for several classes of abelian varieties for which the Mumford-Tate conjecture is known to hold.

In joint work with Samuel Le Fourn and David Zywina, we give an unconditional formula for the optimal exponent β_A, and show that it agrees with the conjectured one under the assumption of Mumford-Tate. Using this, it is easy to prove that the conjecture of Hindry and Ratazzi is in fact equivalent to the Mumford-Tate conjecture for abelian varieties over number fields.

TIME: Wednesday 28, 9:30-10:20.
ROOM: Salle de conférence.
SPEAKER: **Daniel Bertrand** (Paris, France).
TITLE: *Variations on a CM theme.*
ABSTRACT: The relative Manin–Mumford conjecture can be viewed as a local to global principle. In the case of relative dimension 2, it fails in essentially only one case (Ribet sections), which involves an elliptic curve with complex multiplications and two divisors satisfying certain conditions. The recent work of Masser and Zannier on elementary integrability can also be viewed as a local to global property, and it too fails under the very same conditions. We will look for a tentative connection between these two failures, or at least between their proofs.

TIME: Wednesday 28, 10:50-11:40.
ROOM: Salle de conférence.
SPEAKER: **Stéphane Fischler** (Orsay, France).
TITLE: *Linear independence of values of polylogarithms.*
ABSTRACT: Given a positive integer a and a fixed non-zero algebraic number z with $|z| < 1$, it is conjectured that 1 and the values $\mathrm{Li}_k(z) = \sum_{n=1}^{\infty} z^n/n^k$ for $1 \leq k \leq a$ are linearly independent over the rationals. In this lecture we will discuss partial results in this direction, when z is fixed and a tends to infinity. Using explicit constructions, Rivoal and Marcovecchio have obtained a lower bound on the dimension of the subspace spanned over $\mathbb{Q}$ by these numbers. This bound can be improved using Siegel's lemma, and has also been adapted (in a joint paper with Rivoal) to the case where z lies outside the disk of convergence. A motivation for these results is that values at positive integers of the Riemann ζ function are obtained for $z = 1$.

TIME: Thursday 29, 9:30-10:20.
ROOM: Salle de conférence.
SPEAKER: **Julie Desjardins** (Toronto, Canada).
TITLE: *Density of the rational points on a family of del Pezzo surfaces of degree 1*
ABSTRACT: Let X be an algebraic variety over a number field k. We want to study the set of k-rational points $X(k)$. For example, is $X(k)$ empty? If not, is it dense with respect to the Zariski topology? Del Pezzo surfaces are classified by their degrees d, an integer between 1 and 9. Manin and various authors proved that for all del Pezzo surfaces of degree > 1 is dense provided that the surface has a k-rational point (that lies outside a specific subset of the surface for $d = 2$). For $d = 1$, the del Pezzo surface always has a rational point. However, we don't know in general if the set of rational points is Zariski-dense. In this talk, I present a result, joint with Rosa Winter, in which we prove the density of rational points for an interesting family of del Pezzo surfaces of degree 1 over k.

TIME: Thursday 29, 10:50-11:40.
ROOM: Salle de conférence.
SPEAKER: **Amilcar Pacheco** (Rio de Janeiro, Brazil).
TITLE: *On the rank of abelian varieties over function fields.*
ABSTRACT: Let k be a field, and K/k a one variable function field of genus γ. Let A/K be an abelian variety of dimension g. We shall discuss first upper bounds for the rank of the Lang-Néron group of A with respect to K/k, in terms of g, γ and the conductor $f_{A/K}$ of A/K. Next we shall discuss the validity of the Lang-Néron theorem, if we take an infinite Galois extension L of K, *i.e.*, whether the Lang-Néron group of A with respect to L/k is finitely generated. Finally, we shall give sufficient conditions for this to happen in the case of a pro-ℓ extension L of K, where $\ell \neq p = \mathrm{char}(k)$ is a prime number. In particular, we shall discuss an example in which these sufficient conditions are satisfied.

TIME: Thursday 29, 14:00-14:50.
ROOM: Salle de conférence.
SPEAKER: **Christian Maire** (Besançon, France).
TITLE: *On Galois representations with large image.*
ABSTRACT: Let $G_{\mathbb{Q}}$ be the absolute Galois group of $\mathbb{Q}$. For every prime number $p \geq 3$ and every integer $m \geq 1$, we show the existence of a continuous Galois representation $\rho : G_{\mathbb{Q}} \to Gl_m(\mathbb{Z}_p)$ which has open image and is unramified outside $\{p, \infty\}$ (resp. outside $\{2, p, \infty\}$) when $p \equiv 3$ mod 4 (resp. $p \equiv 1$ mod 4). To do that, we combine some properties of pro-p-extensions of number fields with restricted ramification, and lifting mod p Galois representations in terms of embedding problems. During this lecture I will mention the context (this is a result of 2021) and some recent extensions. If time permits, I will also talk about Galois representations unramified at p; this last section is a work in progress.

TIME: Thursday 29, 15:20-16:10.
ROOM: Salle de conférence.
SPEAKER: **Michael Tsfasman** (CNRS, IITP, and IUM).
TITLE: *Asymptotic distribution of Frobenius roots for curves and abelian varieties over a finite field.*
ABSTRACT: We are interested in the behaviour of Frobenius roots when the base field is fixed and the genus of the curve or the dimension of the abelian variety tends to infinity. I shall explain how to put the question and what are the answers. For curves (and for number fields) these are my old results with Serge Vladuts, for abelian varieties those of J.-P. Serre (séminaire Bourbaki, 2018) and my work in progress with Nicolas Nadirashvili.

TIME: Thursday 29, 16:20-17:10.
ROOM: Salle de conférence.
SPEAKER: **Sinnou David** (Paris, France).
TITLE: *Small points and unit groups*
ABSTRACT: We discuss progress towards conjectures of Bertrand and Villegas on arithmetic properties of subgroups of units of a number field, suggesting that one can expect lower bounds interpolating from the question of Lehmer (one dimensional subgroups) all the way up to lower bounds for the regulator (full unit group).

TIME: Friday 30, 9:30–10:20.
ROOM: Salle de conférence.
SPEAKER: **Wojtek Gajda** (Poznań, Poland).
TITLE: *On Mumford-Tate conjecture.*
ABSTRACT: I'll discuss some results of my recent joint work with Marc Hindry. We have checked the Mumford-Tate conjecture for several new classes of abelian varieties defined over number fields.

TIME: Friday 30, 10:50-11:40.
ROOM: Salle de conférence.
SPEAKER: **Florian Breuer** (Newcastle, Australia).
TITLE: *Modular polynomials for elliptic curves and Drinfeld modules.*
ABSTRACT: I will give an overview of some results, old and new, about modular polynomials, both in the classical case of elliptic curves, as well as in the analogous case of Drinfeld modules.

TIME: Friday 30, 14:00-14:50.
ROOM: Salle de conférence.
SPEAKER: **Richard Griffon** (Clermont-Ferrand, France).
TITLE: *The Brauer–Siegel conjecture for elliptic curves*
ABSTRACT: A few years ago, Hindry conjectured that sequences of Abelian varieties satisfy a statement resembling the Brauer–Siegel theorem. The conjecture predicts that the product of the Néron–Tate regulator by the order of the Tate–Shafarevich group has a tame asymptotic behaviour in terms of the height as the latter grows. Except for some general partial results and a handful of sequences for which this asymptotic behaviour has been completely elucidated, the conjecture remains largely open.
In this talk, we will focus mainly on sequences of elliptic curves over function fields in positive characteristic. After introducing the relevant objects, I will start by recalling the statement of Hindry's conjecture in this context, as well as some of the motivations behind it. I will then survey some of the known results towards this conjecture, and briefly sketch their proofs and explain the techniques involved.
Time permitting, I will moreover mention more general statements of "Brauer–Siegel type" for sequences of other arithmetico-geometric objects.